

ECE 443/643 Homework 3

September 22, 2011

1. Let $G(f) = \Lambda(f/B)$ and $x(t) = \Re(g(t)e^{2\pi j f_c t})$, with $B \ll f_c$. *Note: $\Re(z)$ is the real part of z .*
 - (a) Find $x(t)$.
 - (b) Sketch $X(f)$.
2. Let $x(t)$ be bandlimited ($X(f)$ is negligibly small for $|f| > B$). A first order lowpass system has the transfer function $H(f) = \frac{1}{1+j\frac{f}{B}}$. Show that a first-order lowpass LTI system yields approximately distortionless transmission if $x(t)$ is bandlimited to $W \ll B$.
3. Let $x(t) = \Pi(t/W)$, with $W < T$. Find the Fourier series of $g(t) = \sum_{n=-\infty}^{\infty} x(t - nT)$. *Note: $g(t)$ is a square wave with duty cycle W/T .*
4. Let the output of a system $y(t) = 2x(t) - 3(x(t))^3$.
 - (a) Show that the system is time-invariant.
 - (b) Let $x(t) = A \cos(\omega_0 t)$. Find the Fourier series of $y(t)$.
5. *ECE643 only.* Let

$$x(t) = x(t - T) = \begin{cases} -1, & -\frac{T}{2} < t < -\frac{t_f}{2} \\ \frac{2t}{t_f}, & -\frac{t_f}{2} < t < \frac{t_f}{2} \\ 1, & \frac{t_f}{2} < t < \frac{T}{2} \end{cases}, \quad 0 \leq t_f < T$$

($x(t)$ is T -periodic). t_f is a constant.

- (a) Sketch $x(t)$.
- (b) Find the Fourier series coefficients of $x(t)$.
- (c) Comment on $X[k]$ compared to the Fourier series of a 50% duty cycle square wave. In particular, how does $|X[k]|$ go to zero as k gets large?